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EMMA PREVIATO AND HER MATHEMATICAL LIFE (1952-2022)

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In June 2022 we organized a mathematics conference in Vlora, Albania titled *Recent trends in Algebra, Geometry, and Arithmetic*. Professor Emma Previato was one of the many illustrious mathematicians who were scheduled to speak at the conference. Her talk was titled *Polynomial equations are solvable in terms of modular forms*, which was from a joint project Emma had with the second author of this note. Sometime during the first week of June 2022 we got an email from Emma saying that she was ill and would not be able to attend the conference. The conference was during June 9-12, 2022. About a week later, around June 20, Emma wrote us another email congratulating us for a successful conference and how she wished she was there. It was the last exchange we had with her. She passed on June 29, 2023. This special issue of the Albanian Journal of Mathematics in memory of Emma Previato celebrates and honor her life and mathematical achievements.

A summary of her work and life achievement are two volumes proceedings edited by Ron Donagi and Tony Shaska in celebration of her 65-th birthday [20, 21] and especially [19]. This note is an extension of [19] with addition of our own memories of having the privilege to know Emma and possibly being some of the very last mathematicians to have communicated with her before her passing.

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Emma received a Bachelor's degree from the University of Padua in Italy, and a PhD from Harvard University under the direction of David Mumford in 1983. Her thesis was on hyperelliptic curves and solitons. The work on hyperelliptic curves has evolved and expanded into Emma's life-long interest in algebraic geometry.

Emma Previato worked in different areas, using methods from algebra, algebraic geometry, mechanics, differential geometry, analysis, and differential equations. The bulk of her research belongs to *integrable equations*. She is noted for often finding unexpected connections between integrability and many other areas, often including various branches of algebraic geometry.

As an undergraduate at the University of Padua, Italy, Emma wrote a dissertation on group lattices, followed by several publications [74–79]. With methods from algebra, initiated by Dedekind in the 19th century, this area's goal is to relate the group structure to the lattice of subgroups, and provide classifications for certain properties: an excellent overview is the article by Freese [36], a review of the definitive treatise by R. Schmidt, where results from all of Emma's papers are used to give one example, a lattice criterion for a finitely-generated group to be solvable.

Emma's Ph.D. thesis [81], submitted at Harvard in 1983 under the supervision of David Mumford [80] in the area of *integrable equations*, which grew and unified disparate parts of mathematics over the next twenty years, and is still very active. Emma's original tool for producing exact solutions to large classes of nonlinear PDEs, the Riemann theta function, remained one of her main interests.

She later pursued more theoretical aspects of *special functions*, such as Prym theta functions [40, 87, 101, 129, 130] also surprisingly related to numerical results in conformal field theory, the Schottky problem [64], and Thetanulls [73]. She continuously returned to theta functions. In her collaboration with Wijesiri and the second editor of this volume [73] they find relations among thetanulls for genus 3 curves with extra automorphisms and study relations among the classical thetanulls of cyclic curves, namely curves $\mathcal{X}$ (of genus $g(\mathcal{X}) > 1$) with an automorphism σ such that σ generates a normal subgroup of the group G of automorphisms, and $g(\mathcal{X}/\langle\sigma\rangle) = 0$. Relations between thetanulls and branch points of the projection are the object of much classical work, especially for hyperelliptic curves, and of recent work, in the cyclic case. Further work on thetanulls was pursued in [6] and [95].

The area of integrable PDEs is surprisingly related to *algebraically completely integrable Hamiltonian systems*, or ACIS, in the sense that *algebro-geometric* solutions of integrable hierarchies linearize on Abelian varieties, which can be organized into angle variables for an ACIS over a suitable base, typically a subset of the moduli space of curves whose Jacobian is the fiber [82, 87]. Thanks to this discovery, the area integrates with classical geometric invariant theory, surface theory, and other traditional studies of algebraic geometry. With the appearance of the moduli spaces of vector bundles and Higgs bundles over a curve, at the hands of N. Hitchin in the 1980s, large families of ACIS were added to the examples, as well as theoretical algebro-geometric techniques. In [2, 40, 83, 84], Emma took up the challenge of generalizing the connection between ACIS and integrable hierarchies to curves beyond hyperelliptic. In [41], the families of curves are organized as divisors in surfaces.

On the PDE side, the challenges were of two types. When the ring of functions on the (affine) spectral curve can be interpreted as differential operators with a higher dimensional space of common eigenfunctions, the fiber of the integrable system is no longer a Jacobian: it degenerates to a moduli space of higher-rank vector bundles, possibly with some auxiliary structures [102]. Neither the PDEs nor the integrable systems have been made explicit in higher rank in general. Some cases, however, are worked out in [48–50, 65, 103]. The other challenge is to increase the dimension of the spectral variety, for example from curve to surface. Despite much work, this problem too has arguably no explicit solution in general. An attempt to set up a general theory over a multi-dimensional version of the formal Universal Grassmann Manifold of Sato which hosts all linear flows of solutions of integrable hierarchies, is given in [51], and more concrete special settings are mentioned below, under the heading of *Differential Algebra*.

Coverings of curves an important aspect of theta functions is their reducibility, a property whose investigation goes back to Weierstrass and his student S. Kowalevski. Given their special role in integrability, reducible theta functions are invaluable for applied mathematicians to approximate solutions, or even derive exact expressions and periods in terms of elliptic functions. To the algebro-geometric theory of Elliptic Solitons, initiated by I.M. Krichever and developed by A. Treibight and his thesis supervisor J.-L. Verdier, Emma contributed [16, 29, 31, 66, 85, 88, 92, 101], while [22, 72] generalize the reduction to hyperelliptic curves or Abelian subvarieties. More general aspects of elliptic (sub)covers are taken up in [1], where some of the results of the Ph.D. thesis and later work of the second author are summarized [104–107, 118, 119].

Another type of special solution is the one obtained by *self-similarity* [3]; the challenge here is to find an explicit relationship between the PDE flows and the deformation in moduli that obeys Painleve-type equations: this is one reason why Emma's work has turned to a special function which is associated to Riemann's theta function but only exists on Jacobians: the *sigma function*.

Classical theorems of projective geometry can be generalized to ACIS [90, 93], while the challenge of matching them with integrable hierarchies is still ongoing [44]. Explicit Hamiltonians for the Hitchin system are only available in theory: they are given explicit algebraic expression in [131] (cf. also [67], which led to work on the geometry of the moduli space of bundles [63]). An explicit integration in terms of special functions leads to the problem of *non-commutative theta functions* [95].

Differential Algebra is younger than Algebraic Geometry, but it has many features in common. Mumford gives credit to J.L. Burchnall and T.W. Chaundy for the first *spectral curve*, the Spectrum of a commutative ring of differential operators [89]. This is arguably the reason behind algebro-geometric solutions to integrable hierarchies. On the differential-algebra setting, Emma published [8, 86], connecting geometric properties of the curve with differential resultants, a major topic of elimination theory which is currently being worked out [42, 43] and naturally leads to the higher-rank solutions: their Grassmannian aspects are taken up in [23–26, 100] the higher-dimensional spectral varieties are addressed in [97]. Other aspects of differential algebra are connected to integrability in [96] (the action of an Abelian vector field on the meromorphic functions of an Abelian variety) and [9] (a p-adic analog); in [35], the deformations act on modular forms.

Klein extended the definition of the (genus-one) Weierstrass sigma function to hyperelliptic curves and curves of genus three. H.F. Baker developed an in-depth theory of PDEs satisfied by the hyperelliptic sigma function, which plays a key role in recent work on integrable hierarchies (KdV- type, e.g.). Beginning in the 1990s, this theory of Kleinian sigma functions was revisited, originally by V.M. Buchstaber, V.Z. Enolskii and D.V. Leykin, much extended in scope, eventually to be developed for telescopic curves (a condition on the Weierstrass semigroup at a point). We go beyond the telescopic case in [45,46], while we investigate the higher-genus analog of classical theorems in [27,28,30,56,58–60,68] and their connections with integrability in [61] and [57], which gives the first algebro-geometric solutions to a dispersionless integrable hierarchy. It is not a coincidence that its integrable flow on the Universal Grassmann Manifold "cut across" the Jacobian flows of traditional hierarchies, and this is where the two variables of the sigma function (the Jacobian, and the modular ones) should unite to explain the mystery of the Painleve"s equations.

Emma was the only co-author of Robert Accola. In their paper [1] the authors give a complete survey of genus two curves which have a degree n cover to an elliptic curve. The problem is quite old and goes back to Legendre, Jacobi, Picard, et al. The authors describe the moduli space of genus two curves that admit a degree n elliptic subcover in several ways: by algebra, group theory, monodromy, and topology. The work of Lange, Kuhn, Shaska, Volklein, Frey, Kani and others on this topic is described in detail. It is an excellent survey of this old and still interesting problem and it masterfully combines all the different approaches of describing the moduli spaces of genus 2 curves covering elliptic curves. The authors give applications of such moduli spaces to the study of integrable systems. For another survey of this same topic one can check [107], where a more computational viewpoint is given. Coincidentally, this topic is the PhD thesis of the third editor.

Emma's primary contribution to this area is through mentoring undergraduate and graduate thesis or funded-research projects. In fact, this research strand began at the prompting of students in computer science who asked her to give a course on curves over fields of prime characteristic, which she ran for years as a vertically-integrated seminar. Together with her PhD student Drue Coles, she published research papers pursuing Trygve Johnsen"s innovative idea of error-correction for Goppa codes implemented via vector bundles [14,15,99], then she pursued overviews and extensions of Goppa codes to surfaces [17].

With her PhD student Caleb Shor, the third author of this note, she pursued towers of function fields and the construction of the corresponding Goppa codes; see [124] which led to several other papers [111–114,125,126].

Emma edited or co-edited four books [18,34,91,94]. In addition to book and journal publication, Emma published reviews (BAMS, SIAM), entries in mathematical dictionaries or encyclopaedias, teaching manuals and online research or teaching materials; she also published on the topic of mentoring in the STEAM disciplines.

Emma contributed a great deal to mathematics outside her area of research as well. She was tirelessly devoted to her students at Boston University, graduate and undergraduate alike. In recognition of her efforts in the classroom, Emma was awarded the annual Mathematical Association of America Northeastern Section Award for Distinguished University Teaching in 2003. As an academic advisor, she oversaw seven PhD theses, five Masters theses, and thirteen undergraduate theses for distinction. For multiple terms, Emma served as Director of Graduate Studies for the Department of Mathematics and Statistics at Boston University.

Emma made it a priority to support students at all levels and backgrounds, providing them with opportunities to engage in research outside of the classroom and to present their results. She regularly advised research projects individually and in groups, often reaching out to students to get them involved. It was common for her research groups to involve a mix of mathematics graduate students, graduate students from other disciplines, and undergraduate students. These collaborations often bore fruit. For example, a few computer science graduate students approached Emma in the early 2000s to learn some algebraic geometry as it applies to the theory of error-correcting codes. Emma invited a few mathematics graduate students, including the third editor of this special issue, to join this research group. A few years later this third editor, with Emma as his advisor, received his PhD in mathematics having successfully completed and defended his dissertation on certain families of algebraic geometry codes. As a bonus, following in Emma's footsteps he developed a true appreciation for an incredibly hot cup of coffee – an appreciation that exists to this day.

Outreach for student groups was important to Emma as well. She founded and advised the Student Chapters of the Mathematical Association of America and of the Association for Women in Mathematics at Boston University. She served as faculty advisor for numerous symposia including AFRAMATH, an annual outreach symposium, and RUMBUS, a annual symposium highlighting research done by undergraduates. Emma was active in mentorship in the community as well, serving as a mentor in Boston area public schools. As an example, she volunteered for the Focus on Mathematics program, traveling to Arlington, Massachusetts, to mentor middle school students working on mathematical exploration projects.

In 2007 when we started the Albanian Journal of Mathematics, Emma was one of the few mathematicians who was supportive and encouraging. She was on the editorial board of the Journal since its inception until the day of her passing. Her commitment to encouraging mathematics in non-traditional places was to be admired. In the Summer of 2008, when we organized a *NATO Advanced Study Institute* in Vlora, Albania, Emma was one of the main speakers and visited Vlora for two weeks together with many other notable mathematicians such as Vera Pless, Kay Magaard, William Hofman, Igor Shparkinski, Sergey Shpectorov, and many others. Emma was ever-present in the conferences that we organized and special sessions of the American Mathematical Society. With her impressive body of work and commitment to all aspects of the discipline, Emma has inspired younger mathematicians at all levels.

We feel privileged to have know Emma as a mathematician and as a person. She was a friend and a colleague and provided inspiration and advise to us and many other younger generations. She will be remembered fondly by her students, colleagues, and many mathematicians who had the chance to know her.

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