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CT BURST ERROR WEIGHT ENUMERATOR OF ARRAY CODES

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Abstract. Recently, CT burst errors originally defined for block codes have been generalized to CT burst errors for array codes [6]. In order to establish a Rieger's type bound for array codes with respect to CT burst errors. Here, we introduce a CT burst error weight enumerator whose coefficients represent the number of CT burst errors of a particular weight. The method of obtaining the CT burst error weight enumerator is obtained by generating function like approach and it does not involve solving equations as presented in [6].

1. Introduction

In classical algebraic coding theory, block codes and their properties have been investigated intensively. On the other hand, array codes have proven to be a good resource for burst error correction. A burst error definition for block codes is given in [1]. Burst error definitions for array codes as two dimensional objects differ. Recently, Jain in [7] has introduced a type of burst error for two dimensional arrays. Later, a new approach on enumerating for these type of errors is introduced in [9]. Recently, Jain in [6] has further generalized this definition for array codes and named these burst errors as CT (Chien-Tang) burst errors in [6]. Jain has investigated these array codes with respect to newly introduced non Hamming metric called Rosenbloom-Tsfasmann (or shortly RT) metric and established a Rieger's type bound. In [6], enumeration of CT burst errors is based on solving some linear equations and further for each weight computation the computations have to be carried out separately. Here in this paper, we introduce a novel approach for computing the number of CT bursts that avoids solving equations and separate computations. We introduce so called CT burst error weight enumerator and the way how to obtain it. The coefficients of CT burst error weight enumerator give the number of CT burst errors of a particular weight. This approach avoids solving equations and repetition of computations.

Definition 1.1. *A linear subspace of $Mat_{m\times s}(F_q)$ (the set of all $m \times s$ matrices over the finite field with q elements) is called an array code.*

Key words and phrases. Matrix Array Codes, Non Hamming Metric, CT Burst Errors, Weight enumerator.

Definition 1.2. [6] *A CT burst of order pr or* $p \times r$ $(1 \le p \le m, 1 \le r \le s)$ *in the space* $Mat_{m\times s}(F_q)$ *is an* $m \times s$ *matrix in which all the nonzero entries are confined to some* $p \times r$ *submatrix which has non zero first row and column.*

Definition 1.3. *(Non Hamming-RT weight)*[10]
Let $\boldsymbol{v} = (v_1, v_2, \ldots, v_n) \in \mathbb{F}_q^n$. *The RT weight (or* ρ*-weight) of* $\boldsymbol{v}$ *is defined by*

$$w_N(\boldsymbol{v}) = \begin{cases} max\{i|v_i \neq 0\}, & \boldsymbol{v} \neq 0 \\ 0, & \boldsymbol{v} = 0. \end{cases}$$

Let $A \in Mat_{m\times s}(F_q)$, and A_i be the ith row of the matrix A. Then the RT weight of the matrix A is the sum of the RT weight of its rows in other words $w_N(A) = \sum_{i=1}^{m} w_N(A_i)$.

The RT (non Hamming) metric for codes over fields is defined in [10] and some bounds for the minimum distance are established. Some applications of this metric to uniform distributions are given in [11]. Some recent work related to RT metric has appeared in [2],[4],[5].

Let $T_{m\times s}^{p\times r}(F_q)$ be the number of CT bursts of order *pr*. This number with a direct computation is given in the following theorem.

Theorem 1.1. [6]

$$T_{m\times s}^{p\times r}(F_q) = \begin{cases} ms(q-1), & p=1, r=1, \\ m(s-r+1)(q-1)q^{r-1}, & p=1, r\ge 2 \\ (m-p+1)s(q-1)q^{p-1}, & p\ge 2, r=1, \\ (m-p+1)s(q-1)q^{r(p-1)}[q^r - 1 - (q^{r-1}-1)q^{1-p}], & p\ge 2, r\ge 2. \end{cases}$$

Further, in [6] a formula for the number of CT bursts of a particular order and ρ-weight less than or equal to a number is stated and proved in the following theorem. It is also shown that this theorem enables to establish a Rieger's type bound for array codes with respect to CT burst errors.

Theorem 1.2. [6] *The number of CT bursts of order pr* $(1 \le p \le m, 1 \le r \le s)$ *in* $Mat_{m\times s}(F_q)$ *having* ρ*-weight* w *or less* $(1 \le w \le ms)$ *is given by*

$$T_{m\times s}^{p\times r}(F_q, w) = \begin{cases} m \times \min(w,s) \times (q-1), & p=1, r=1, \\ m \times \min(w-r+1, s-r+1) \times (q-1)q^{r-1}, & p=1, r\ge 2, \\ (m-p+1)T_3, & p\ge 2, r=1, \\ (m-p+1)T_4, & p\ge 2, r\ge 2. \end{cases}$$

where

$$T_3 = \sum_{j=1}^{\min(w,s)} \sum_{\eta=0:\eta j\le w-j}^{p-1} \binom{p-1}{\eta} (q-1)^{\eta+1},$$

$$T_4 = \sum_{j=1}^{\min(w-r+1,s-r+1)} (Q_{j,r}^{p} - Q_{j,r}^{p-1} - Q_{j+1,r-1}^{p} + Q_{j+1,r-1}^{p-1}),$$

and

$$(1) \qquad Q_{j,r}^{p} = \sum_{k_j,k_{j+1},\ldots,k_{j+r-1}} \frac{p!q^{\sum l=0^{r-1}(l+1)k_{j+l}}}{\prod_{l=0}^{r-1} k_{j+1}! \left(p - \sum_{l=0}^{r-1} k_{j+l}\right)!} \left(\frac{q-1}{q}\right)^{\sum_{l=0}^{r-1} k_{j+l}}$$

where $k_j, k_{j+1}, \ldots, k_{j+r-1}$ being nonnegative integers such that

$$\sum_{l=0}^{r-1} k_{j+l} \leq p$$

$$\sum_{l=0}^{r-1} (j+l)k_{j+l} \leq w. \tag{2}$$

In Theorem 1.2, computing the number of CT burst errors of a particular order is still a challenging task. In Equation 2, the two inequalities are first to be solved in the set of natural numbers, then by using these k_i solutions the numbers $Q^p_{j,r}$ are to be computed by the formulas in (1) and finally after having found the necessary values, the formula in Theorem 1.2 is applied. In [6], some examples using this approach are worked out explicitly. In the next sections, we introduce a new method that is simpler than the method introduced in [6] and explained above. Further, by using the new method, it does not only give the number of a particular CT burst error weight but it also gives all spectra of the weights in a single computation. The spectra of the number of burst errors shall be called the burst error weight enumerator.

In Section 2, we give the computation method of burst errors of order $p \times r$ in the space $Mat_{p\times r}(F_q)$. In Section 3, we give the computation method of burst errors of order $p \times r$ in the space $Mat_{m\times s}(F_q)$ where $1 \leq p \leq m, 1 \leq r \leq s$ by making use of the results obtained in Section 2. We conclude by several remarks.

2. Burst Error Weight Enumerator

In order to introduce the new counting approach for $T^{p\times r}_{m\times s}(F_q)$ of CT burst errors we need couple definitions.

We shall work on the space $Mat_{p\times r}(F_q)$ and consider only burst errors of order $p \times r$. Later, we shall consider burst errors of order $p \times r$ in the larger space $Mat_{m\times s}(F_q)$ where $1 \leq p \leq m, 1 \leq r \leq s$.

In this section, first we introduce the concept of generic burst errors. Next, we present the method of computing the number of generic burst errors. Then, we introduce a method for computing the number of burst errors by making use of generic burst errors.

Definition 2.1. *If $A \in Mat_{m\times s}(F_q)$ and $w_N(A_i) = \alpha_i$, then the matrix A is said to have a weight distribution of type $(\alpha_1, \alpha_2, \ldots, \alpha_m)$.*

Definition 2.2. *(Hamming weight) If an element of F_q is not equal to zero than its Hamming weight is equal to 1, otherwise 0. $w(a) = 1$ if $a \neq 0$ or else $w(a) = 0$. Hamming weight of a codeword is the sum of Hamming weights of its coordinates.*

Definition 2.3. *A generic burst error $A = (a_{ij})$ of order $p \times r$: A burst error of order $p \times r$ with the following conditions:*

1. *All entries are equal to* 0 *or* 1.
2. *If the first entry of a row is nonzero then the Hamming weight of that row is equal to* 1 *or* 2.
3. *If the first entry of a row is zero then the Hamming weight of that row is equal to* 1.

Let $A = (a_{ij}) \in Mat_{p\times r}(F_q)$ and A_i be the ith row of the matrix A. Let $A_i = (a_{i1}, a_{i2}, \dots, a_{ir})$. Since in our method of representing a matrix as a multivariable term and also carrying information for the first column entries of the matrix is crucial we associate a multivariable term $x_i^{w(a_{i1})} X_i^{w_N(A_i)}$ where $w(a_{i1}) = 1$ if $a_{i1} \neq 0$ and $w(a_{i1}) = 0$ if $a_{i1} = 0$. We use capital letter variables for the entries different from the first entry and small letter variable for the first entry only. In a natural way, we extend this representation to the matrix A by taking the product of all terms corresponding to the rows of A.

For example, the representation of the following matrices are given below:

Example 2.1.

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \leftrightarrow \quad X_1^2 x_2 X_3^3, \quad B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \quad \leftrightarrow \quad x_1 X_3^2.$$

Since there is a correspondence between p multivariable polynomials and generic burst errors of order $p \times r$, it is possible to list these errors via multivariable polynomials.

For simplification purpose, we set $(x_1, \dots, x_p, X_1, \dots, X_p) = (\tilde{x}, \tilde{X})$.

Theorem 2.1. *Let $\overline{X}_j = 1 + \sum_{i=2}^r X_j^i$ for all $1 \leq j \leq p$. All generic bursts of order $p \times r$ are obtained as terms of the following multi variable polynomial, say generic polynomial:*

$$G(\tilde{x}, \tilde{X}) = x_1 \overline{X}_1 \prod_{j=2}^r (1 + x_j)\overline{X}_j + \left(\overline{X}_1 - 1\right) \sum_{j=2}^r \frac{x_j \prod_{i=2}^r (1 + x_i)\overline{X}_i}{(1 + x_j)}.$$

Proof: Let $A = (a_{ij}) \in Mat_{p\times r}(F_q)$ be a generic burst error. We split the proof into two parts. First, if $a_{11} = 1$, then the corresponding p-variable terms must all contain the multiples of x_1 of order one. Thus, for the first row all possible terms that contain x_1 and a power of X_2 greater than one are represented by $x_1 \left(1 + \sum_{i=2}^r X_j^i\right)$. Since there is no restriction on the rest of terms, these are all obtained from the terms $x_1 \overline{X}_1 \prod_{j=2}^r (1 + x_j)\overline{X}_j$. In the second case, if $a_{11} = 0$, then, by definition of a generic burst, there must exist $a_{1j} = 1$, for some $2 \leq j \leq p$. Since the term x_1 does not exist, any multiple of the terms in the sum $\sum_{i=2}^r X_1^i = \overline{X}_1 - 1$ can exist and further at least one of x_j $(j \geq 2)$ must exist. Hence, the corresponding p-variable terms are obtained from the terms of $\left(\overline{X}_1 - 1\right) \sum_{j=2}^r \frac{x_j \prod_{i=2}^r (1+x_i)\overline{X}_i}{(1+x_j)}$. Therefore, by adding these two group of possible terms, we have the result. $\square$

Example 2.2. *By applying Theorem 2.1, the following generic polynomial G gives the term representation of $A \in M_{2\times 3}(F_2)$ generic burst errors of order 2×3 : $(X_1 = X, X_2 = Y, x_1 = x, x_2 = y)$*

$$G(x, y, X, Y) = x(1 + X^2 + X^3)(1 + y)(1 + Y^2 + Y^3) + (X^2 + X^3)y(1 + Y^2 + Y^3).$$

Hence,

$$G(x,y,X,Y) = x + xY^2 + xY^3 + xy + xyY^2 + xyY^3 + xX^2 + xX^2Y^2 + xX^2Y^3 + xX^2y + xX^2yY^2 + xX^2yY^3 + xX^3 + xX^3Y^2 + xX^3Y^3 + xX^3y + xX^3yY^2 + xX^3yY^3 + X^2y + X^2yY^2 + X^2yY^3 + X^3y + X^3yY^2 + X^3yY^3.$$

Theorem 2.2. *Let $G(x_1,\dots,x_p,X_1,\dots,X_p)$ be the generic polynomial of generic bursts of order $p\times r$. Let $x_1^{a_1}x_2^{a_2}\cdots x_p^{a_p}X_1^{b_1}X_2^{b_2}\cdots X_p^{b_p}$ be a term of the generic polynomial G where $a_i=0$ or $a_i=1$ and $2\le b_i\le r$. Then, by substituting*

$$(q-1)^{\sum(a_i+b_i)}q^{\sum\max(b_i-2,0)}x_1^{a_1}x_2^{a_2}\cdots x_p^{a_p}X_1^{b_1}X_2^{b_2}\cdots X_p^{b_p}$$

for $x_1^{a_1}x_2^{a_2}\cdots x_p^{a_p}X_1^{b_1}X_2^{b_2}\cdots X_p^{b_p}$, we obtain a multivariable polynomial, say K. Next, by substituting $X_j^{c_j}$ for $x_jX_j^{c_j}$ if $c_j\neq 0$ and X_j for x_j otherwise in K, we obtain a multivariable polynomial say $H(X_1,\dots,X_p)$. The coefficients of H corresponding to the term $X_1^{c_1}X_2^{c_2}\cdots X_p^{c_p}$ give the number of all burst errors of order $p\times r$ and type $(c_1,c_2,\dots,c_p)$ in the space $Mat_{p\times r}(F_q)$.

Proof: Let $x_1^{a_1}x_2^{a_2}\cdots x_p^{a_p}X_1^{b_1}X_2^{b_2}\cdots X_p^{b_p}$ be a term of the generic polynomial G. The ith row is determined by the variables x_i and X_i corresponding the first and the last entries. If $a_i=1$, then the first entry of the ith row is not equal to zero. Hence, there are $q-1$ non zero elements in F_q for this entry. Similarly, if $b_j\neq 0$, then the entry (j,b_j) of the burst error is not equal to zero. Hence, there are $q-1$ non zero elements in F_q for this entry, too. The entries between the first and the last entry of the ith row can take any value of F_q. Thus, considering the case $b_i=0$, there are $q^{\max(b_i-2,0)}$ choices for these entries. Altogether, for the ith row, there are $(q-1)^{a_i+b_i}q^{\max(b_i-2,0)}$ choices. Multiplying all terms corresponding to the rows of the burst error, we obtain the coefficients of a polynomial K. Since the term $x_jX_j^{c_j}$ when $c_j\neq 0$ corresponds to the jth row with the first and the c_jth entry nonzero, the ρ weight of the jth row is equal to c_j, hence substituting X_j for the term $x_jX_j^{c_j}$ will protect this information when considered as a new term of a polynomial. On the other hand if $c_j=0$, then only the term x_j will appear, and in this case by substituting X_j for x_j will serve for our purpose. It is clear that after these substitutions, the coefficients of the term $X_1^{c_1}X_2^{c_2}\cdots X_p^{c_p}$ in the new multivariable polynomial say H will give the number of burst errors of order $p\times r$ and type $(c_1,c_2,\dots,c_p)$ in the space $Mat_{p\times r}(F_q)$.

Definition 2.4. *The multivariable polynomial $H(X_1,\dots,X_p)$ obtained in Lemma 2.2 is said to be the weight spectra burst error enumerator of burst errors of order $p\times r$.*

Example 2.3. *Let G be given as in Example 2.2. Then, by making necessary substitutions given in Theorem 2.2, we have*

$$H(X,Y) = XY + 3X^2Y^2 + 6X^2Y^3 + 4X^3Y + 12X^3Y^3 + 2XY^2 + 6X^3Y^2 + X + 2X^2Y + 2X^3 + 4Y^3X + X^2.$$

Definition 2.5. *Let $\mathbb{B}$ be the set of all burst errors of order $p\times r$. The polynomial, $B^{p\times r}(t)=\sum_{A\in\mathbb{B}}t^{w_N(A)}=\sum_{i=1}^{p\cdot r}b_it^i$ is said to be the burst error weight enumerator of bursts of order $p\times r$ in the space $Mat_{p\times r}(F_q)$.*

The following corollary is straightforward:

Corollary 2.1. *Let $H(X_1, \ldots, X_p)$ be the weight spectra burst error enumerator of burst errors of order $p \times r$. By setting $X_1 = X_2 = \cdots = X_p = t$ in $H(X_1, \ldots, X_p)$ we obtain $B^{p\times r}(t)$.*

Example 2.4. *Substituting t for both X and Y in Example 2.2, we obtain the burst error weight enumerator*

$$B^{2\times 3}(t) = 12t^6 + 12t^5 + 11t^4 + 6t^3 + 2t^2 + t.$$

Example 2.5. *Let $\boldsymbol{B}$ be the set of all burst errors of order 4×2 in the space $Mat_{4\times 2}(F_2)$. Let $X_1 = X, X_2 = Y, X_3 = Z$ and $X_4 = W$. Then,*

$$\begin{aligned}
&G(\tilde{x}, \tilde{X}) = x(1+X^2)(1+y)(1+Y^2)(1+z)(1+Z^2)(1+w)(1+W^2) \\
&+ X^2(y(1+Y^2)(1+z)(1+Z^2)(1+w)(1+W^2) + z(1+Z^2)(1+y)(1+Y^2)\cdot \\
&\cdot (1+w)(1+W^2) + (1+y)(1+Y^2)w(1+W^2)(1+z)(1+Z^2)).
\end{aligned}$$

Further, by applying necessary substitutions as pointed out in Corollary 2.1, we obtain the burst error weight enumerator

$$B_{4\times 2}(t) = 20t^8 + 44t^7 + 57t^6 + 52t^5 + 31t^4 + 15t^3 + 4t^2 + t.$$

It is clear that the sum of the coefficients of order three or less it is equal to $T^{4\times 2}_{4\times 2}(F_2, 3) = 20$.

This example is also worked out in [6]. In order to compute the value of $T^{4\times 2}_{4\times 2}(F_2, w)$ for a particular value of w, we need to apply the formula given in [6] for each case separately. However, with this novel approach, we can compute each value of w quite easily by adding the related coefficients of the weight enumerator of burst errors. For instance, $T^{4\times 2}_{4\times 2}(F_2, 4) = 51$. This fact can be formalized easily by the following corollary:

Corollary 2.2.

$$T^{p\times r}_{p\times r}(F_q, w) = \sum_{i=0}^{w} w_i$$

where w_i correspond to the coefficients of burst error weight enumerator.

3. Bursts in Larger Space

In this section we consider burst of errors of order $p\times r$ in the space $Mat_{m\times s}(F_q)$ where $1 \le p < m$ and $1 \le r < s$.

Lemma 3.1. *Let A be a burst of order $p \times r$ in the space $Mat_{m\times s}(F_q)$ where $1 \le p < m$ and $1 \le r < s$. If $T^{p\times r}(F_q)$ is the number of burst of order $p \times r$ in the space $Mat_{p\times r}(F_q)$, then $T^{p\times r}_{m\times s}(F_q) = (s-r+1)(m-p+1)T^{p\times r}_{p\times r}(F_q)$ gives the number of burst of order $p \times r$ in the space $Mat_{m\times s}(F_q)$.*

Proof: A burst of order $p \times r$ in the space $Mat_{m\times s}(F_q)$ is an A submatrix of a matrix in $Mat_{m\times s}(F_q)$ with confined non zero entries in a submatrix of size $p \times r$ with a nonzero first row and column. Hence, placing A as a submatrix of a matrix of size $m \times s$ is possible in $s - r + 1$ ways moving from the left to the right starting from the position $(1, 1)$ for both the matrix and the submatrix. Also, given any possible position obtained above, there exist also $m - p + 1$ movements downwards for obtaining all possible submatrices. Thus, there exist $(s - r + 1)(m - p + 1)$

positions that give raise to new submatrices for each burst error of order $p\times r$ in the space $Mat_{p\times r}(F_q)$. Therefore, the number of burst of order $p\times r$ in the space $Mat_{p\times s}(F_q)$ is $(s-r+1)(m-p+1)T^{p\times r}_{p\times r}(F_q)$.□

We naturally extend the definition of a generic burst error of order $p\times r$ in the space $Mat_{p\times r}(F_q)$ to the space $Mat_{m\times s}(F_q)$. Further, we associate a multivariable term $x_i^{jw(a_{ij})}X_i^{w_N(A_i)}$ where $a_{ik}=0$ for all $1\le k<j$ to the ith row of matrix A. Again, we extend this definition to a matrix as in Section 2.

For example, the representation of the following matrices via polynomial terms are given below:

Example 3.1.

$$A=\begin{bmatrix}0&0&0&0\\0&0&1&0\\0&1&0&0\\0&0&0&1\end{bmatrix}\quad\leftrightarrow\quad x_2^3x_3^2x_4^3,\quad B=\begin{bmatrix}0&0&1&1\\0&1&1&0\\0&1&0&0\\0&0&0&0\end{bmatrix}\quad\leftrightarrow\quad x_1^3X_1^4x_2^2X_2^3x_3^2.$$

Lemma 3.2. *Let A be a burst of order $p\times r$ in the space of matrices $Mat_{p\times r}(F_q)$. Let the term $x_1^{a_1}x_2^{a_2}\cdots x_p^{a_p}X_1^{b_1}X_2^{b_2}\cdots X_p^{b_p}$ represent the burst error A of type*

$(\max(a_1,b_1),\max(a_2,b_2),\dots,\max(a_p,b_p))$. If $s-r+1>0$, then multiplying the term $x_1^{a_1}\cdots x_p^{a_p}X_1^{b_1}\cdots X_p^{b_p}$ by $x_1^{w(a_1)}\cdots x_p^{w(a_p)}X_1^{w(b_1)}\cdots X_p^{w(b_p)}$ gives a new burst error in the space $Mat_{m\times s}(F_q)$.

Definition 3.1. *Let*

$$H(X_1,X_2,\dots,X_p)=\sum_{(i_1,i_2,\dots,i_p)}h(i_1,i_2,\dots,i_p)X_1^{i_1}X_2^{i_2}\cdots X_p^{i_p}$$

be a multi variable polynomial where $h(i_1,i_2,\dots,i_p)\in\mathbb{N}$. Then, we define an operator T on multivariable polynomial H as follows:

$$T(H)=\sum_{(i_1,i_2,\dots,i_p)}h(i_1,i_2,\dots,i_p)X_1^{w(i_1)(i_1+1)}X_2^{w(i_2)(i_2+1)}\cdots X_p^{w(i_p)(i_p+1)}.$$

Example 3.2. *We consider the burst errors of order 2×2 in the space of matrices $Mat_{3\times 3}(F_2)$. Then,*

$$G(\tilde{X},\tilde{Y})=x(1+X^2)(1+y)(1+Y^2)+X^2(y(1+Y^2)).$$

and

$$G(\tilde{X},\tilde{Y})=x+xY^2+xy+xyY^2+xX^2+xX^2Y^2+xX^2y+xX^2yY^2+X^2y+X^2yY^2.$$

There are 10 burst errors of order 2×2 in the space of matrices $Mat_{2\times 2}(F_2)$.

There is $s-r=1$ movement to the right and obtaining 10 more bursts in the space $Mat_{3\times 3}(F_2)$. Hence, 20 burst errors in $Mat_{3\times 3}(F_2)$. Since, $m-p=1$, there is one movement down. Thus, obtaining 20 more burst errors in $Mat_{3\times 3}(F_2)$. Altogether, there are $(s-r+1)(m-p+1)\times 10=4\times 10=40$ matrices in $Mat_{3\times 3}(F_2)$. These burst errors are explicitly listed in Example 3.1 in [6].

$$\begin{aligned}H(X,Y)&=2X^2Y+3X^2Y^2+2Y^2X+X^2+YX+X\\&=(2X+3X^2)Y^2+(2X^2+X)Y+X+X^2.\end{aligned}$$

$$T(H)=2X^3Y^2+3X^3Y^3+2Y^3X^2+X^3+Y^2X^2+X^2.$$

$H+T(H) = 2Y^2X+4X^2Y^2+2X^2Y+YX+2Y^3X^2+3Y^3X^3+2Y^2X^3+X+2X^2+X^3$. *Setting* $X = Y = t$ *in* $H + T(H)$, *we obtain* $3t^6 + 4t^5 + 4t^4 + 5t^3 + 3t^2 + t$. *Thus,*

$$W_{3\times 3}^{2\times 2}(t) = 6t^6 + 8t^5 + 8t^4 + 10t^3 + 6t^2 + 2t.$$

Hence, the number of burst errors of ρ*-weight* 3 *or less is equal to* $T_{3\times 3}^{2\times 2}(F_2, 3) = 18$.

4. Conclusion

The work presented here is an approach for enumerating burst errors. This generator function like approach can be applied to similar problems. Two dimensional burst error concept in array codes is still an interesting problem. Depending on the definition of burst errors in two dimensional arrays, enumeration of them in order to establish bounds on parameters of codes is an important problem.

References

[1] R.T. Chien and D.T. Tang, *On Definition of a Burst,* IBM Journal Research Development, 9, p.292-293, 1965.

[2] Steven T. Dougherty and Maxim M. Skriganov, *MacWilliams Duality and the Rosenbloom-Tsfasman Metric*, Moscow Mathematical Journal, Vol. 2 Number 1, p. 83-89, 2002.

[3] P. Fire, *A class of Multiple Error Correcting Binary Codes for Non-independent Errors,* Sylvania Reports RSL-E-2, Sylvania Reconnaissance Systems, Mountain View, California, 1959.

[4] Mehmet Ozen, Irfan Siap, *On The Structure And Decoding of Linear Codes with respect to Rosenbloom-Tsfasman Metric*, Selcuk Journal of Applied Mathematics, Vol. 5, No. 2, pp.25-31,2004.

[5] Mehmet Ozen, Irfan Siap, *Linear codes over* $\mathbb{F}_q[u](u^s)$ *with respect to the Rosenbloom-Tsfasman metric*, Designs Codes and Cryptography, Vol.38, p. 17-29, 2006.

[6] Sapna Jain, *CT Bursts- From Classical to Array Coding,* Discrete Mathematics, 308(9), p.1489-1499, 2008.

[7] Sapna Jain, *Bursts in m- Metric Array Codes,* Linear Algebra and Its Applications, Vol. 418, p.130-141, 2006.

[8] Irfan Siap, *The Complete Weight Enumerator for Codes over* $\mathcal{M}_{n\times s}(\mathbb{F}_q)$, Lecture Notes on Computer Sciences 2260, p. 20-26, 2001.

[9] Irfan Siap, *Burst Error Enumeration of m-Array Codes,* submitted, 2007.

[10] M. Yu Rosenbloom and M. A. Tsfasman, *Codes for the m-metric,* Problems of Information Transmission, Vol. 33. No. 1, 45-52, 1997.

[11] M.M. Skriganov, *Coding theory and uniform distributions,* St. Petersburg Math. J. Vol 143, No. 2, 2002.